B.Sc. Part—I Semester—I Examination
MATHEMATICS
(Algebra & Trigonometry)
Paper—I

Time : Three Hours] [Maximum Marks : 60

1. (1) If $f(\lambda) = 0$ is the characteristic equation of the matrix A, then $f(A) =$ _____.
   - (a) 0
   - (b) 1
   - (c) –1
   - (d) None

   (2) The characteristic root of a non-singular matrix is :
   - (a) Non-zero
   - (b) Zero
   - (c) One
   - (d) None

   (3) The value of $|3 + i4| =$ ______.
   - (a) 5
   - (b) 4
   - (c) 7
   - (d) None

   (4) Real part of $\sin(x + iy) =$ ____.
   - (a) $\sin x \cdot \cosh y$
   - (b) $\cos x \cdot \sinh y$
   - (c) $\sin x$
   - (d) None

   (5) If $a + \sqrt{b}$ is a root of the equation $f(x) = 0$ with integral coefficient then _____ is also the roots of equation where $a, b \in R$ and b is positive :
   - (a) $a / \sqrt{b}$
   - (b) $\sqrt{a} / b$
   - (c) $a - \sqrt{b}$
   - (d) None

   (6) Number of negative real roots for equation $2x^7 - x^4 + 4x^3 - 5 = 0$ is :
   - (a) One
   - (b) Two
   - (c) Three
   - (d) Zero

   (7) The series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ... + (-1)^{n-1} \frac{x^{2n-2}}{(2n-2)!} + ...$, $-\infty < x < \infty$ is called :
   - (a) cosine series
   - (b) sine series
   - (c) exponential series
   - (d) None

   (8) The value of $\tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239} =$ _____ .
   - (a) 1
   - (b) π
   - (c) $\pi/2$
   - (d) $\pi/4$

(9) A quaternion q whose real part is zero then it is :

(a) Zero quaternion (b) Unit quaternion

(c) Pure quaternion (d) None

(10) Product of two unit quaternion is :

(a) Zero (b) Unit quaternion

(c) Zero quaternion (d) One 1×10=10

UNIT—I

2. (a) State the DeMoivre's theorem. Prove it for positive integers. 1+3

(b) Separate into real and imaginary parts of tan (x + iy). 4

(c) Prove that $\cosh^2 z - \sinh^2 z = 1$. 2

OR

3. (p) Prove that $\sinh^{-1} x = \log [x + \sqrt{x^2 + 1}]$. 4

(q) If tan (A + iB) = x + iy. Prove that $\tan 2A = \dfrac{2x}{1 - x^2 - y^2}$ and $\tanh 2B = \dfrac{2y}{1 + x^2 + y^2}$. 4

(r) Show that :

$$(\sin x + i \cos x)^n = \cos\left(\frac{n\pi}{2} - n\pi\right) + i \sin\left(\frac{n\pi}{2} - nx\right).$$ 2

UNIT—II

4. (a) If $-\pi/4 \le x \le \pi/4$ then prove that :

$$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x - ....$$ 5

(b) Prove that :

$$\pi = 2\sqrt{3}\left[1 - \frac{1}{3^2} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + ...\right]$$ 5

OR

5. (p) Prove that :

$$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \pi/4$$ 5

(q) Sum the series :

$\cos x \cdot \cos x + \cos^2 x \cdot \cos 2x + \cos^3 x \cdot \cos 3x + ...$ 5

UNIT—III

6. (a) Let p and q be any quaternions then prove that :

$$pq \neq qp \quad \forall \; p, q \in H, \; p \neq q$$ 5

(b) For any p, q $\in$ H show that pq = qp if and only if p and q are parallel. 5

OR

7. (p) For any p, q $\in$ H prove that :

$$(pq)^* = q^*p^*$$ 5

(q) Let quaternions p = 2 – i + 3j – 4k, and q = 5 + 2i – 4j + 3k. Find product pq. 5

UNIT—IV

8. (a) Solve the equation $x^3 - 21x = 344$ by Cardon's method. 5

(b) Solve $x^3 - 12x^2 + 39x - 28 = 0$ its roots being in arithmetical progression. 5

OR

9. (p) State Descarte's rule of sign. Find the nature of roots of the equation

$$3x^4 + 12x^2 + 5x - 4 = 0$$ 2+3

(q) Solve the equation $x^3 - 11x^2 + 34x - 24 = 0$ two roots of which are in the ratio 3 : 2. 3

(r) Determine the value of the $\Sigma\alpha^2$ for the cubic equation $x^3 + px^2 + qx + r = 0$ whose roots are α, β, γ. 2

UNIT—V

10. (a) Prove that eigenvalues of Hermitian matrix are all real. 5

(b) Find the eigenvalues and the corresponding eigenvectors of the matrix :

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ 5

OR

11. (p) Find the row rank and column rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$. 5

(q) Show that an eigenvalue of an idempotent matrix are either zero or unity. 5